Energy Release Rate of Postbuckled Laminates with a Through-Width Delamination

S. K. Cheong* and C. T. Sun**

(Received October 12, 1992)

The strain energy release rate is calculated for buckled one-dimensional delamination (through-width delamination) in composite laminates subjected to in-plane compression. A crack closure method based on plate finite elements is used in this analysis. For some laminates containing a one-dimensional delamination in cylindrical bending, closed form solutions are available. The present finite element solutions show excellent agreement with the analytical solutions. The strain energy release rate for various types of laminates is also calculated using the present finite element method. The results show that the strain energy release rate strongly depends on the type of laminate.

Key Words: Delamination, Crack Closure Method, Strain Energy Release Rate, One-Dimensional Delamination

1. Introduction

Delamination can reduce the load-carrying capacity of composite structures. When the delaminated composite structure is under compressive loading, postbuckling of the delaminated region may create high interlaminar stresses and extensive delamination growth, which leads to structural instability. Since the stresses at the delamination boundary are mathematically oscillating (England, 1965 ; Erdogan, 1963 ; Williams, 1959), calculated stresses there have little meaning. Strain energy release rates are finite parameters which characterize the intensity of the stresses near the singularity. Hence, many of the current efforts to predict instability-related delamination growth are based on strain energy release rates (Chai, 1981 ; Yin and Wang, 1984 ; Whitcomb, 1981 ; Yin, 1985 ; Chai and Babcock, 1985 ; Yin and Jane, 1989 ; Whitcomb and Shivakumar, 1989 ; Whitcomb, 1989).

One-dimensional delamination (i.e., through-width delamination) (see Fig. 1) was analyzed on the basis of beam-plate theory (Chai, 1981 ; Yin and Wang, 1984). Chai et al.(1981) calculated the strain energy release rate by differentiation of the strain energy with respect to delamination length. The strain energy was calculated by assuming the generalized plane strain condition. For simplicity the properties of the plate were assumed homogeneous, isotropic, and linearly elastic. Yin and Wang(1984) used the path-independent J-integral concept to obtain an expression of the energy release rate for a homogeneous orthotropic plate. The energy release rate was expressed in terms of the axial forces and bending moments acting across the various cross sections adjacent to the tip of delamination. The plane strain assumption was used in that paper. But the incorrect buckling strain was used in that paper (Yin and Wang, 1984). Whitcomb(1981) analyzed the postbuckled through-width delamination on the basis of geometrically nonlinear finite element stress analysis. The stress distributions and strain energy

* Department of Aerospace Engineering, Chosun University, 375 Seosuk-dong, Dong-gu, Kwangju 501-759, Korea
** School of Aeronautics and Astronautics, Purdue University, W. Lafayette, IN 47907, U.S.A.

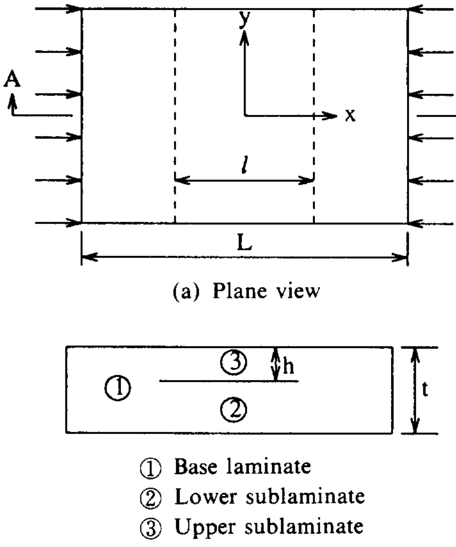


Fig. 1 Configuration of a through-width delamination

release rates were calculated for various delamination lengths, delamination depths, applied loads, and lateral deflections. From the preceding papers (Chai, 1981 ; Yin and Wang, 1984 ; Whitcomb, 1981), the strain energy release rate can be calculated only for delamination in isotropic plate and unidirectional composite plate.

In this study, a crack closure method based directly on plate resultants and kinematic variables is proposed for calculating the strain energy release rate of postbuckled delamination. By using the plate finite elements, the strain energy release rate can be calculated for one-dimensional delamination in various types of laminated composite plates. The present method can be applied to two-dimensional delamination problem. Two-dimensional delamination problem will be presented in the next paper. Geometrical nonlinearity in the buckled region is taken into consideration. The strain energy release rate is calculated incrementally. The accuracy of this method is evaluated by comparing the present solution with existing closed form solutions for one-dimensional delamination problems. The strain energy release rate for various types of laminates is also calculated using the present finite element method.

2. Analytical Solutions

Accurate postbuckling solution of a through-the-width delamination (see Fig. 1) may be obtained relatively easily. Both general and local postbuckling solutions for the through-the-width delamination are available (Chai, 1981 ; Yin and Wang, 1984).

Chai et al.(1981) solved a through-the-width delamination problem based on beam-column theory. For the general case of through-the-width delamination, eight equations were obtained, which can not be solved in closed form. Cylindrical bending of the plate was assumed along with a generalized plane strain condition. A numerical iterative scheme was employed to calculate the energy release rate. A closed form solution for a thick beam model was obtained by neglecting bending contributions of the base laminate and lower sublaminate. The formula for G^*, that is, Eq. (27) of Chai et al.(1981) can be rewritten as follows :

$$G^* = \frac{(1-\nu^2)Eh}{2(1-\bar{h}+\bar{h}\bar{l})^2}(1-\bar{h})(\varepsilon_o^* - \varepsilon_{cr}^*) \left[\varepsilon_o^* + \varepsilon_{cr}^*\left(3+4\frac{\bar{h}\bar{l}}{1-\bar{h}}\right)\right], \tag{1}$$

where $\bar{h}=h/t$, $\bar{l}=l/L$, and

$$\varepsilon_{cr}^* = \frac{\pi^2}{3(1-\nu^2)}\left[\frac{h}{l}\right]^2, \tag{2}$$

is the buckling strain for the upper sublaminate.

Yin and Wang(1984) solved a through-the-width delamination problem by using the J-integral method. They considered a homogeneous orthotropic plate of a linearly elastic material whose orthotropic axes coincide with the longitudinal, normal, and transverse directions of the plate (x_1, x_2, x_3, respectively). The solution of a general case with the plane strain condition was obtained in terms of the axial forces and bending moments as follows (Yin and Wang, 1984) :

$$G = \frac{1-\nu_{13}\nu_{31}}{2E_1t^3}\left[\frac{(tP^*)^2}{\bar{h}(1-h)} + \frac{12(M^*)^2}{\bar{h}^3} + \frac{12(tP^*/2-M^*)^2}{(1-\bar{h})^3}\right], \tag{3}$$

where

$$P^* = \bar{h}[P_1 + 6(1-\bar{h})M_1/t] - P_3, \tag{4}$$

$$M^* = M_3 - M_1\bar{h}^3, \tag{5}$$

where P_1 and M_1 are axial force and bending moment of the base laminate ahead of the crack tip. The axial force and bending moment of the upper sublaminate behind the crack tip are P_3 and M_3, respectively.

In this study, the compliance method is used to obtain the expression for the strain energy release rate for a similar problem but with the plane strain condition. Considering P_b as a force acting on the base laminate, the strain energy is

$$U = L\int_0^{\varepsilon_o} P_b d\varepsilon = L\int_0^{\varepsilon_{cr}} P_b d\varepsilon + L\int_{\varepsilon_{cr}}^{\varepsilon_o} P_b d\varepsilon. \tag{6}$$

Assuming the plane strain condition, we can obtain the buckling strain as follows.

$$\varepsilon_{cr} = \frac{\pi^2}{3}\left[\frac{h}{l}\right]^2. \tag{7}$$

Assume that the upper sublaminate is thin as compared with the base laminate and the lower sublaminate, and P_b can be expressed as follows :

$$P = \frac{Et}{(1-\nu_2)}\varepsilon_o (0 \le \varepsilon_o < \varepsilon_{cr}), \tag{8a}$$

$$P_b = \frac{Et}{(1-\nu^2)}\frac{(1-\bar{h})\varepsilon_o + \bar{h}\bar{l}\varepsilon_{cr}}{(1-\bar{h}+\bar{h}\bar{l})}(\varepsilon_o \ge \varepsilon_{cr}). \tag{8b}$$

Equation (8b) was obtained by calculating the axial force of the base laminate from Eq.(25) of Chai et al.(1981) and dividing by $(1-\nu^2)$ to account for the plane strain assumption. From the above equations, the strain energy can be obtained as follows :

$$U = \frac{LEt}{(1-\nu^2)}\left[\frac{\varepsilon_{cr}^2}{2} + \frac{1}{(tL-hL+hl)}[L(t-h)\frac{(\varepsilon_o^2-\varepsilon_{cr}^2)}{2} + hl\varepsilon_{cr}(\varepsilon_o-\varepsilon_{cr})]\right]. \tag{9}$$

The strain energy release rate is expressed as follows :

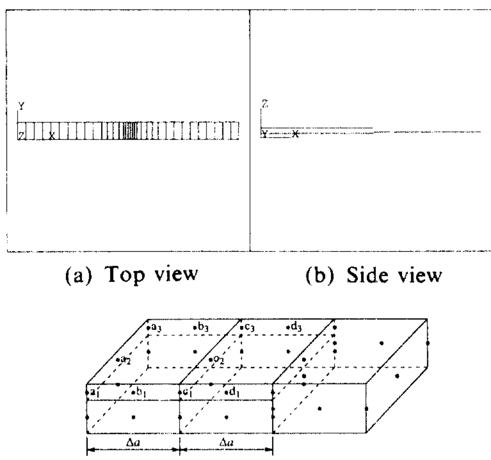
$$G = -\frac{\partial U}{\partial l} = \frac{Eh}{(1-\nu^2)}\frac{1}{2(1-\bar{h}+\bar{h}\bar{l})^2}(1-\bar{h})(\varepsilon_o-\varepsilon_{cr})\left[\varepsilon_o + \varepsilon_{cr}[3+4\frac{\bar{h}\bar{l}}{1-\bar{h}}]\right]. \tag{10}$$

3. Finite Element Modelling and Crack Closure Method

ANSYS (a commercial finite element code) was used to analyze the configurations considered. An eight-noded layered plate element (Kohnke, 1989) was used in the study. This element includes the effect of transverse shear deformation (Ahmad, 1970). Geometrical nonlinearity resulting from large deflection was included in the analysis.

In the finite element analysis, the upper and lower sublaminates in the delaminated region and the base plate were modelled by plate elements. At the delamination front, the nodes were rigidly connected to ensure continuity in displacements and rotations.

Figure 2 shows the finite element model for a one-dimensional delamination problem. Figure 2(c) shows the configuration of the delamination front (at nodes c_i) before the virtual crack extension. The crack closure concept requires letting the crack grow by Δa and then using the nodal forces and moments at nodes c_i and d_i before crack extension to close the opening due to the extensions. If Δa is small, this crack opening can be approximated by the crack opening at nodes a_i



(a) Top view (b) Side view

(c) Configuration of delamination front region

Fig. 2 Finite element model for a one-dimensional delamination and b_i before crack extension. Since postbuckling of the delaminated region is nonlinear, an incremental crack closure procedure was used.

Assume that the upper sublaminate is thin and would buckle first. The strain energy release rate can be calculated approximately by using the following formula. For the upper sublaminate, we have

$$G=\frac{1}{\Delta A}\sum_{ls=1}^{LS}\sum_{j=1}^{3}\sum_{i=1}^{5}(F_i^{cj}\Delta u_i^{aj}+F_i^{dj}\Delta u_i^{bj}), \tag{11}$$

where

ΔA = crack closure area,
u_i^{aj} = i-th component of displacement (i=1, 2, 3) or rotation (i=4, 5),
Δu_i^{aj} = increment of u_i^{aj} between load steps,
F_i^{cj} = i-th component of force (i=1, 2, 3) or moment (i=4, 5) at node c_j,
LS = number of load steps.

As seen from Fig. 2(c), the terms related to b_2 and d_2 should be excluded in Eq. (11). To obtain the total strain energy release rate the contributions of the lower sublaminate should be added to Eq. (11).

4. Results and Discussion

In the numerical example, both isotropic materials and composites were considered. For isotropic materials, the elastic properties were given by

E=67 GPa (9.7 msi), ν=0.33.

The material properties of the graphite/epoxy used in this study were

E_1=134 GPa (19.4 msi),
$E_2=E_3$=10.2 GPa (1.48 msi),
$G_{12}=G_{13}$=5.52 GPa (0.8 msi),
G_{23}=3.43 GPa (0.5 msi),
$\nu_{12}=\nu_{13}$=0.3,
ν_{23}=0.49.

The thickness of each ply is 0.127 mm (0.005 inch).

The dimensions of the beam model were

L=203.2 mm (8 inches),
ℓ=101.6 mm (4 inches),
t=5.08 mm (0.2 inch),
h=1.016 mm (0.04 inch).

These dimensions correspond to 8 graphite/epoxy plies for the upper sublaminate and 32 plies for the lower sublaminate.

Figure 3 shows the result of the isotropic thick beam model for the generalized plane strain condition. The closed form solution was obtained by Chai(1981).

Figure 4 shows the results for the general case of a through-the-width delamination for the

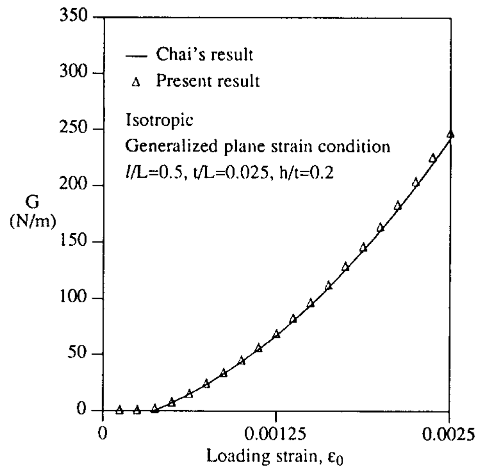


Fig. 3 Energy release rate for a thick beam model of a postbuckled through-width delamination

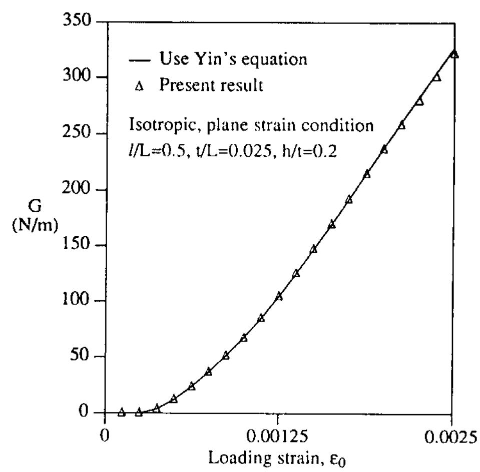


Fig. 4 Energy release rate for a general case of a postbuckled through-width delamination

isotropic material. The analytical solution was obtained by substituting the moments and forces obtained from the finite element analysis into Eq. (3).

Figure 5 shows the results for the general case of a postbuckled laminate $[0_8//0_{32}]$. The symbol (//) in $[0_8//0_{32}]$ laminate indicates that the delamination is between 0_8 plies and 0_{32} plies. Again the analytical solution was obtained from Eq. (3) using the moments and forces obtained from the present finite element analysis.

The above results clearly indicate that the present finite element crack closure method is quite accurate in calculating the strain energy release rate for this type of problem.

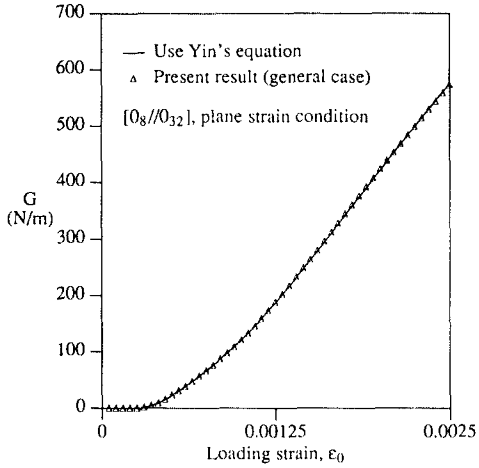


Fig. 5 Energy release rate for a postbuckled laminate $[0_8//0_{32}]$

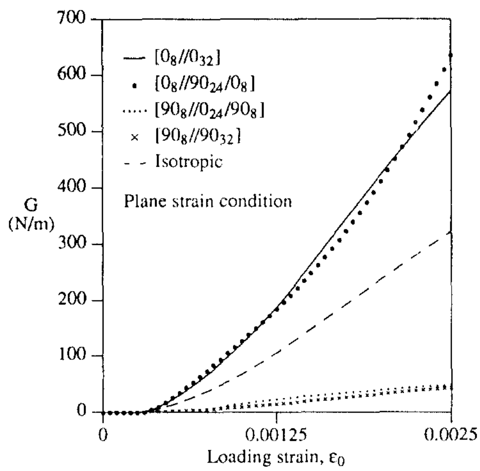


Fig. 6 Energy release rate for various postbuckled laminates

Figure 6 shows the results for various kinds of postbuckled laminates. The results show that the energy release rates strongly depend on the type of laminate. Analyzing the postbuckled through-width delamination, twenty load steps and fifty load steps were used for the isotropic plate and the composite plate, respectively.

5. Conclusions

An incremental crack closure method in conjunction with plate finite elements was used to calculate strain energy release rate for one-dimensional delamination subjected to in-plane compression. The finite element solutions for various kinds of delamination problems were compared with analytical solutions. Excellent agreement was obtained.

The strain energy release rate for one-dimensional delamination in various types of laminated composite plates was also calculated using the present finite element method. The results show that the strain energy release rate strongly depends on the type of laminate. The strain energy release rate for laminate $[0_8//90_{24}/0_8]$ is about thirteen times that for laminate $[90_8//0_{24}/90_8]$ when the applied strain is 0.0025.

The present method can be also extended to calculate the strain energy release rate for two-dimensional delamination in various types of laminated composite plates.

Acknowledgements

The author would like to acknowledge the support of Chosun University.

References

Ahmad, S., 1970, "Analysis of Thick and Thin Shell Structures by Curved Finite Elements," Int. J. Numerical Methods in Engineering, Vol. 2, pp. 419~451.

Chai, H. and Babcock, C. D., 1985, "Two-

Dimensional Modelling of Compressive Failure in Delaminated Laminates," J. Composite Materials, Vol. 19, pp. 67~98.

Chai, H. et al., 1981, "One Dimensional Modelling of Failure in Laminated Plates by Delamination Buckling," Int. J. Solids & Structures, Vol. 17, No. 11, pp. 1069~1083.

England, A. H., 1965, "A Crack between Dissimilar Media," J. Appl. Mech., Vol. 32, pp. 400~402.

Erdogan, F., "Stress Distribution in a Nonhomogeneous Elastic Plane with Cracks," J. Appl. Mech., Vol. 30, pp. 232~236.

KohnKe, P. C., 1989, ANSYS Theoretical Manual, Swanson Analysis Systems Inc.

Whitcomb, J. D., 1981, "Finite Element Analysis of Instability Related Delamination Growth," J. Composite Materials, Vol. 15, pp. 403~426.

Whitcomb, J. D. and Shivakumar, K. N., 1989, "Strain-Energy Release Rate Analysis of Plates with Postbuckled Delaminations," J. Composite Materials, Vol. 23, No. 7, pp. 714~734.

Williams, M. L., 1959, "The Stresses around a Fault or Crack in Dissimilar Media," Bull. Seism. Soc. Am., Vol. 49, pp. 199~204.

Yin, W. L. and Jane, K. C., 1989, "Postbuckling Analysis of Laminated Elliptical Plates with Application to Delamination Growth," Proc. AIAA/ASME/ASCE/AHS/ASC 30th SDM Conference, Mobil, Alabama, pp. 121~134.

Yin, W. L. and Wang, J. T. S., 1984, "The Energy-Release Rate in the Growth of a One-Dimension Delamination," J. Appl. Mech., Vol. 51, pp. 939~941.